



Modeling Gerotor Oil Pumps in 1D to Predict Performance with Known Operating Clearances

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ABSTRACT

Engine and transmission oil systems are commonly pressurized by gerotor style pumps, due to their simplistic design and low cost. Gerotor pumps are designed with certain tolerances of the gears and housing, thus creating a tradeoff of lower cost with larger tolerances and higher cost with smaller tolerances. By building a detailed gerotor pump model with a 1D hydraulic flow network, engineers can evaluate pump performance with these tolerances as input and compare to find the optimal design. This paper showcases the ease of building a gerotor model in 1D by using an automated process extracting the key model inputs directly from the pump CAD file. The gerotor pump performance is predicted including flow rate, total power loss, volumetric efficiency, and total efficiency vs. pump speed, pressure rise, clearance tolerances, and temperature, and compared with experiment. A predictive friction model for gerotor pumps is proposed and total torque required to drive the pump is compared directly with experiment. Additionally, an internal leakage model for predicting volumetric efficiency taking into account physical clearance between inner and outer gears, as well as temperature is discussed.

CITATION: Harrison, J., Aihara, R., and Eisele, F., "Modeling Gerotor Oil Pumps in 1D to Predict Performance with Known Operating Clearances," *SAE Int. J. Engines* 9(3):2016, doi:10.4271/2016-01-1081.

INTRODUCTION

Gerotor style pumps have been studied and modeled extensively in the past. Several authors have written about modeling gerotor pumps in 1D. For example, Neyrat et al [1] discussed modeling a transmission gerotor pump with AMESim 1D flow simulation software and predicting the onset of cavitation, as well as predicting fluid borne noise. Fabiani et al [2] developed a kinematic relationship for the gear set to calculate volume vs. rotation angle, and also used AMESim 1D software to validate this approach. Inaguma [3] came up with a mathematical model for calculating friction torque in a gerotor pump, though it is unclear if the model can be extended to general use. Montero and Codina utilized a bond graph approach to modeling the gerotor pump performance and pressure ripple [4-5] and also developed a detailed leakage model considering various leakage paths in the pump [6]. Frosina et al [7] developed a gerotor pump model with AMESim which correlated well to experimental pressure pulsations and also incorporated an oil system downstream of the pump to predict pulsations throughout the system. Ruvalcaba and Hu [8] looked at the influence of tip-to-tip clearance variation on volumetric efficiency and compared 3D CFD predictions in ANSYS FLUENT with experimental data, though the authors do not offer a general rule of volumetric efficiency or leakage flow as a function of clearance.

To the author's knowledge, no work has been presented discussing how to model predictively the leakage in a 1D flow model based on known experimental clearance tolerances, or use fundamental friction relations to calculate total power consumption of the pump based on known clearances. These topics will be the focus of this paper.

MODELING GEROTOR PUMPS IN 1D

SHW is a German company that produces pumps for passenger vehicles and Truck and Off-Highway applications. A schematic of the SHW gerotor pump that will be examined in this paper is shown in [Figure 1](#). The inner gear is the drive gear which is driven from a shaft, and the outer gear is the driven gear. In a gerotor the inner gear has one less tooth than the outer gear so the outer gear rotates slower than the inner gear by the ratio of inner gear teeth / outer gear teeth. [Figure 5](#) shows a typical 1D flow network representation of the gerotor pump shown in [Figure 1](#). Since it takes the outer gear one revolution to complete the full gerotor pumping cycle, the number of pumping chambers is equal to the number of teeth on the outer gear. The volume of these chambers change as the pump rotates and they are also communicating with an inlet volume and outlet volume through port area profiles. [Figure 1](#) only shows one volume for each inlet and outlet side for simplicity, but gerotor pumps often have inlet and outlet ports on both sides of the gears.

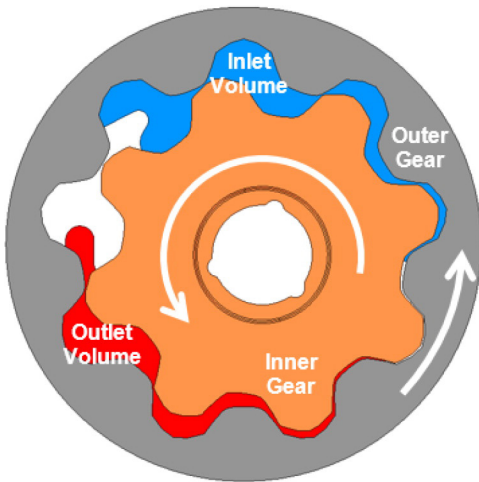


Figure 1. Schematic of an SHW gerotor pump.

In the 1D flow model using GT-SUITE simulation software, the pump flow system is discretized into many volumes comprised of pipes and flowsplits (volumes), and pipe passages are further discretized into subvolumes. The 1D Navier-Stokes equations, namely conservation of mass, momentum and energy are solved every timestep in every subvolume. Conservation of species is also considered to solve for species transport throughout the system. This is very important in a gerotor as pump performance can be significantly affected by aeration content or cavitation. In Figure 2, the 1D staggered grid approach used by the solver is shown, where scalar quantities like pressure, internal energy, temperature, density, etc. are solved at the center of each subvolume, and scalar quantities like mass flow rate, velocity, species flow rate, etc. are solved at the boundary between subvolumes.

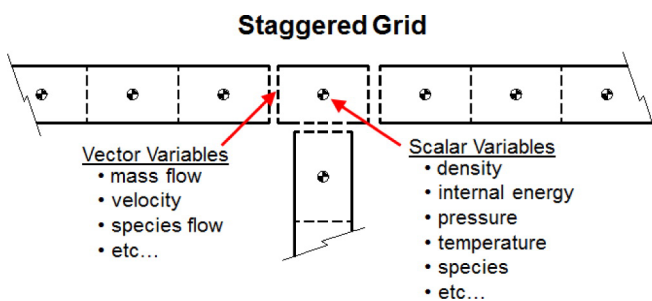


Figure 2. Staggered grid solution utilized in GT-SUITE for solving Navier-Stokes equations and species transport in 1D.

At each timestep the Navier-Stokes equations are applied to each subvolume and boundary in the grid, though the order in which the subvolumes are solved is not important. The solver makes no assumption about the transient evolution of the species content in a particular subvolume, rather it solves for the transient mass fraction of each species in each subvolume. A fluid mixture is considered homogeneously mixed within a given subvolume, and consequently flow from one subvolume into another will be based on the species mass fractions in the upstream subvolume. In terms of cavitation, if the pressure in any subvolume falls below the vapor pressure of a given liquid at a given temperature, the solver will iterate to calculate the amount of liquid that has cavitated. This cavitating vapor can then be transported throughout the system.

The model has 30 total flow volumes, 13 of which are pipe subvolumes. The average timestep is 1.4×10^{-6} seconds. Most cases converge in 4 to 5 pump cycles, requiring less than 1 minute of real time to run on a modern PC.

Gerotor Pump Model Building Process

Building gerotor pump models can be time consuming, as a complete model requires the following critical information:

1. Discretizing the entire flow path into subvolumes
2. The volume and each port area profile must be defined
3. All dominant leakage paths must be considered

The pre-processing tool called GEM3D in GT-SUITE enables an easy model building process for gerotor pumps as a CAD model of the gerotor pump can be imported into GEM3D and by pointing to the inner gear, outer gear, and inlet/outlet port shapes, the 1D model will be built automatically for the user. This allows the user to spend little time building the model and more time analyzing the model.

After the 1D model is built, the user will need to define boundary conditions in the flow system, such as initial pressure and fluid properties, as well as the aeration level in the working fluid. The pump speed should also be defined and depending on the type of analysis, a transient profile of the pump speed can be defined but it can also be set to a constant speed for steady state analysis. Figure 3 shows the process of importing the CAD model of the pump into GEM3D, discretizing the flow volumes, and then generating the 1D flow model. Included in the flow model are the automatically generated volume and area profiles as shown in Figure 4.

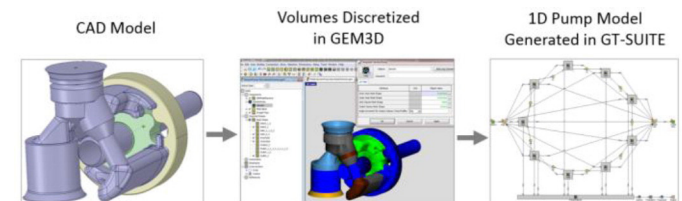


Figure 3. Process of importing CAD model into the GEM3D pre-processor and generating the 1D flow model.

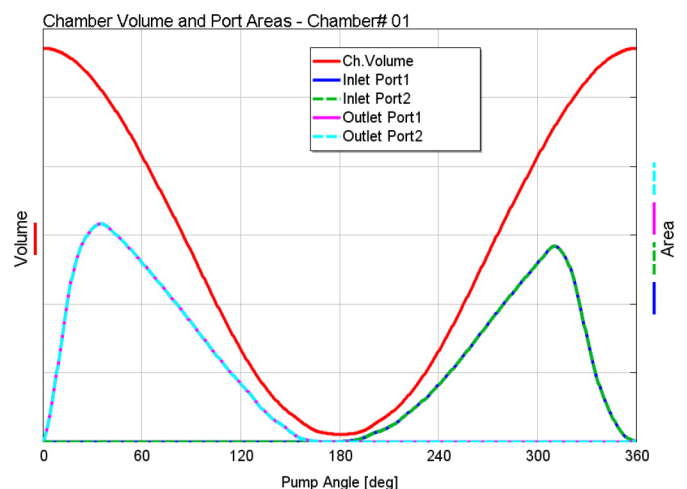


Figure 4. Volume and port area profiles of the SHW pump generated using GEM3D.

Model Setup

A schematic of the model of the SHW transmission oil pump is shown in Figure 5. Since the SHW pump has 9 outer teeth, there are 9 chamber volumes, and one cycle of the pump is represented as one revolution of the outer gear. The flow path from the oil pan to the chamber volumes is highlighted in blue, while the flow path from the chambers to the outlet is highlighted in red. The leakage path between chambers is highlighted in green. The "OilPan" part on the map represents the ambient condition in the oil pan from the test. The changing volume in each "Vol1", Vol2", etc... part pulls the oil from the pan into each chamber as its volume expands. Then when the volume in each chamber compresses, the oil is forced to the outlet side of the pump. The "OutletRestrict" part uses a calibrated orifice diameter to meet the test target pressure of either 2 or 10 bar-gauge. The internal leakage between each chamber is modeled with parts "gap1", "gap2", etc... and the details of this internal leakage is described below.

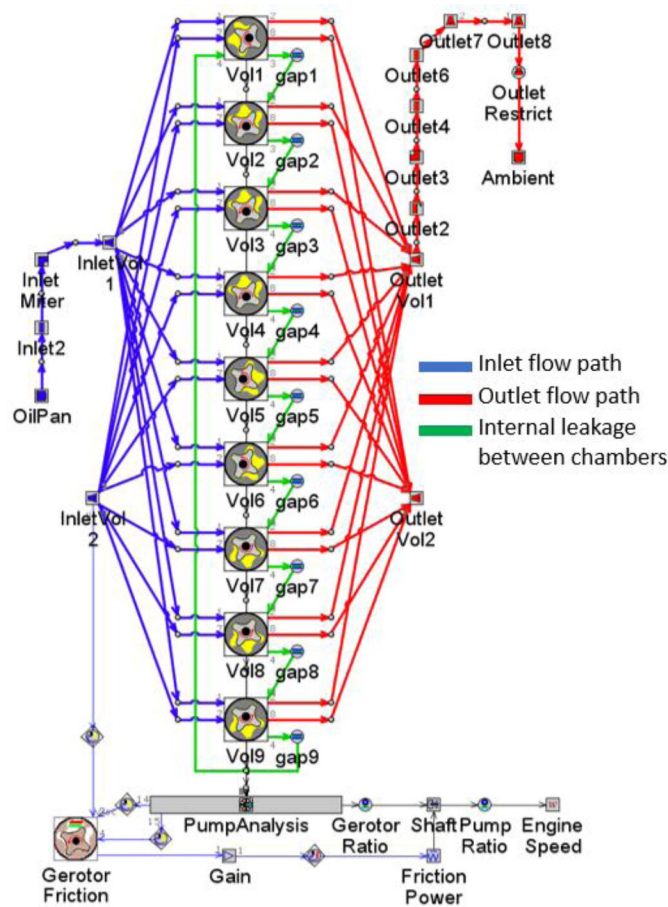


Figure 5. Model map showing all flow paths in the 1D flow model.

Modeling Internal Leakage

Classically speaking, 1D flow models can be used to calibrate the internal leakage to correlate to experimental volumetric efficiency of a gerotor pump by assuming either an orifice equation type of restriction as shown in Equation (1), which does not take viscous effects into account, or using a Poiseuille flow relation in a rectangular duct to characterize the leakage as shown in Equation (2), which requires a

characteristic passage length that is not easy to characterize and is often simply tuned. Furthermore the Reynolds Number through the gap may be large enough to invalidate Poiseuille flow.

$$Q_{orifice} = C_d A \sqrt{\frac{2(P_{1t} - P_2)}{\rho}} \quad (1)$$

$$Q_{Poiseuille} = \frac{W \cdot \Delta P \cdot C_{IGOG}^3}{12 \mu L} \quad (2)$$

The issue with these models is that they do not easily lend themselves to predict the leakage in a robust manner based on the physical clearances between pumping chambers and oil properties. Since the clearance gap between the pumping chambers, C_{IGOG} , is known from test measurement, the model will take advantage of this input to the leakage model. The orifice solution in this model can be considered a hybrid form of Equation (1) and (2) by considering both inertia effects at high Reynolds number and viscous effects at low Reynolds number. The orifice model uses momentum conservation balancing forces on both sides of the orifice. Assuming the flow is steady, laminar, incompressible, with discharge coefficient equal to 1, and neglecting pressure recovery which is an ok assumption when the orifice area is much smaller than the downstream volume, the leakage flow reduces to the following equation:

$$Q_{leak} = A \sqrt{\frac{2(P_{1t} - P_2)}{\rho(1 + C_f \frac{A_s}{A})}} \quad (3)$$

where the cross-sectional area of the leakage path, A , friction factor, C_f and surface area, A_s , are defined below:

$$A = C_{IGOG} W \quad (4)$$

$$C_f = \frac{24}{Re} = \frac{24 \mu A}{\rho Q_{leak} 2 C_{IGOG}} \quad (5)$$

$$A_s = (2 C_{IGOG} + 2 W) L \quad (6)$$

Note the friction factor uses 24 instead of the traditional 16 for laminar flow. This is due to the friction factor of a thin rectangular duct being 50% higher than a circular duct [9]. Also note that assuming the clearance gap of the rectangular duct is much smaller than its width, then if the friction term is much larger than the inertia term in Equation (3), the equation reduces to Equation (2) for Poiseuille flow. Conversely, if the Reynolds number is sufficiently high, then Equation (3) reduces to Equation (1) for orifice flow.

The length of the rectangular leakage, L , was calibrated to best match the test data, and was found to be a function only of the clearance gap between the inner and outer gear, using the following relation:

$$L = -55.55C_{IGOG} + 6.55 \quad (7)$$

Table 1 shows the experimental clearance gap in the pump which is directly used in the leakage flow equation. This gap represents the gap between the inner and outer gear when using a feeler gauge to measure the total play between the gears. The clearance is not changing with temperature because both the inner and outer gear are made of steel and have little thermal expansion relative to each other.

Table 1. Experimental clearance gap between the inner gear and outer gear.

	30°C		125°C	
	Min Clearance	Max Clearance	Min Clearance	Max Clearance
Clearance Gap Between Inner and Outer Gear, C_{IGOG} (μm)	70	100	70	100

It is worth noting that other leakage paths were considered for the study, including between chambers via the sides of the gears and the housing. This leakage was an order of magnitude smaller than the leakage discussed above and was considered negligible enough to be excluded. Additionally, the isentropic heating of the oil in the leakage gap was considered using the relation in Equation (8) as proposed by Koivula et al [10], but the maximum temperature rise, ΔT , was less than 2°C and was not considered important in the range of outlet pressures used in this study. It could be more important for pumps operating at higher pressures though.

$$\Delta T = \frac{\Delta P}{\rho c_p} \quad (8)$$

Modeling Friction

In order to predict total pump power consumption, both fluid pumping work and frictional loss must be predicted. In the model there are four components of friction considered, including:

1. Viscous friction between the inner gear sides and housing
2. Viscous friction between the outer gear sides and housing
3. Viscous friction between the outer gear outer radius and housing
4. Viscous friction at the bearing support

These four components of friction can be seen visually on the faces highlighted red in Figure 6.

The first component is the friction power between the inner gear sides and housing, P_{f1} , which is calculated using the following relation:

$$P_{f1} = 2\mu \frac{A_{IG}\omega_{IG}^2(R_{IG})^2}{c_{IG}} \quad (9)$$

Since there are two sides of the inner gear, there is a 2 in Equation (9). The second component is the friction power between the outer gear sides and housing, P_{f2} , which is calculated using the following relation:

$$P_{f2} = 2\mu \frac{A_{OG}\omega_{OG}^2(R_{OG})^2}{c_{OG}} \quad (10)$$

The third component is the friction power due to the shear in the radial gap between the outer gear and housing, P_{f3} , which is calculated with the following relation assuming no loading in this journal bearing film:

$$P_{f3} = \mu 2\pi W \frac{\omega_{OG}^2(R_{OG})^3}{c_{OGB}} \quad (11)$$

The fourth and final component is the friction power due to the shear in the radial gap at the journal bearing support, P_{f4} , which is calculated with the following relation assuming again no loading in this journal bearing film:

$$P_{f4} = \mu 2\pi W_{brg} \frac{\omega_{IG}^2(R_{brg})^3}{c_{brg}} \quad (12)$$

All four friction components are then summed to arrive at the total friction power, $P_{friction}$, as shown in Equation (13):

$$P_{friction} = P_{f1} + P_{f2} + P_{f3} + P_{f4} \quad (13)$$

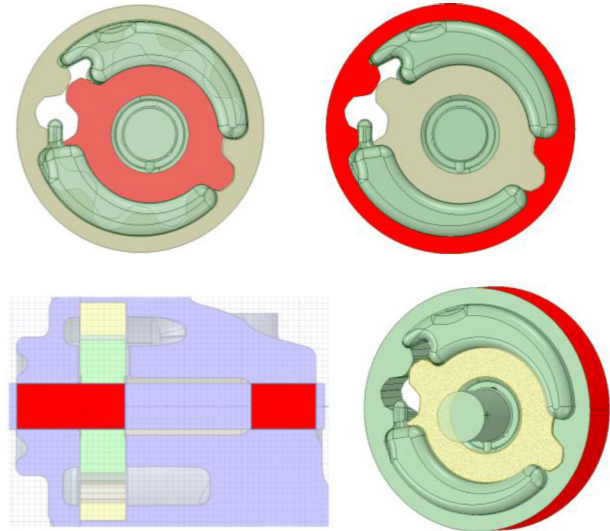


Figure 6. Four components of friction considered in the friction model. Friction interfaces are highlighted red. Starting clockwise from top left, inner gear side, outer gear side, outer gear radial gap, bearing support.

The total power required to drive the pump is the sum of the friction and fluid power, where the fluid power, P_{fluid} , is the sum of the PdV work done by all pumping chambers over the time of one pump cycle, represented in Equation (6):

$$P_{fluid} = \frac{\int_{1cycle} \left(\sum_{i=1}^{#ofChambers} p_i dV_i \right)}{t_{cycle}} \quad (14)$$

$$P_{total} = P_{friction} + P_{fluid} \quad (15)$$

Test Setup

Table 2 shows the actual clearances based on measurements from the experimental pump test rig and used directly in the friction model. All gaps are a function of temperature, due to thermal expansion of the aluminum cast housing relative to the Distaloy AB steel gears.

Table 2. Experimental Clearances in Test Setup.

	30°C		125°C	
	Min Clearance	Max Clearance	Min Clearance	Max Clearance
Gap Between Inner Gear and Housing on One Side, C_{IG} (μm)	12.5	25	18.5	31
Gap Between Outer Gear and Housing on One Side, C_{OG} (μm)	11.5	24	17.5	30
Radial Clearance Between Outer Gear and Housing, C_{OGB} (μm)	26.3	51.3	39	64
Shaft Bearing Radial Clearance, C_{BRG} (μm)	7	14.5	9.5	17

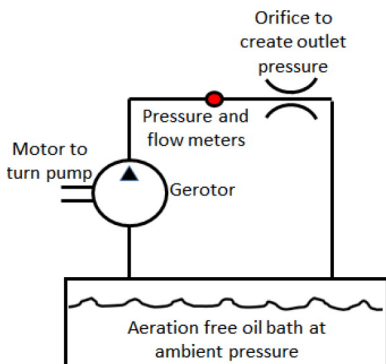


Figure 7. Experimental test setup.

Figure 7 shows a schematic of the test setup, and Table 3 shows the experimental conditions. A motor drives the pump at a given speed. First the torque is measured on the motor before driving the pump in order to establish a tare reading, then the torque is measured again when the pump is operating at a given condition and the tare reading is subtracted out to calculate the torque required by the pump. An orifice on the outlet side of the pump is varied in order to impose the desired outlet pressure. A flow meter and pressure meter are also installed on the outlet side of the pump. Since the pressure meter is installed upstream of the flow meter, the flow meter loss can be bundled into the calibrated outlet restriction orifice in the model. In

the cases compared between model and experiment, the outlet pressure was held at 2 bar and 10 bar-gauge at a given speed, temperature and clearance specification.

Table 3. Experimental conditions of SHW gerotor pump.

Pump Displacement (cc/rev)	2.24
Outlet Pressure (bar-gauge)	2 and 10
Pump Speed (RPM)	500 to 5200
Oil Type	Shell ATF L12108
Oil Temperature (°C)	30, and 125
Oil Kinematic Viscosity (cSt)	36 @ 30°C, and 3.8 @ 125°C
Oil Density (kg/m ³)	839 @ 30°C, and 792 @ 125°C
Air Content (fraction by mass)	0

RESULTS AND COMPARISON

Four sets of experiments were run, namely at low and high oil temperature (30°C and 125°C), with minimum and maximum clearances. Within each graph presented below, there are two outlet pressures shown, at 2 and 10 bar-gauge. The model uses a calibrated orifice diameter to meet the average outlet pressure target, simulating the real orifice in the experiment. The error in the outlet pressure in the model is less than 1% compared to experiment. Figure 8 shows the correlation with flow rate at these four conditions.

The flow correlates well for all conditions with the exception at low speed, high temperature, small clearance, high pressure rise condition. This appears to be a shortcoming of the leakage model, which may require some dependence with speed for better correlation. Furthermore, the Reynolds number through the leakage gap at this condition is on the order of 1500, which is close to the transition region for friction and not as easy to predict as friction in the laminar region, thus potentially causing some uncertainty in the friction factor used in the internal leakage flow Equation (3).

The agreement in flow rate also carries over to the volumetric efficiency prediction in Figure 9.

Figure 10 shows the correlation with total power loss of the pump at these four conditions. The best correlation can be seen at cold temperature with maximum clearances, while the cold temperature with minimum clearances condition overpredicts the total power loss, which points to the friction power being overpredicted because the flow rate is in good agreement at this same condition. One possible reason for the over-prediction in power loss at low temperature and minimum clearance is that in reality the cold oil is being sheared in the thin films by the moving gears, which will cause the fluid to heat up, especially since the fluid recirculates without bulk flow moving in or out of these films. Thus, reality will have a higher oil temperature in these films compared to the model which assumes the temperature is the same as in the oil pan. This effect is most pronounced at this cold oil, minimum clearance, high speed condition. This idea could be investigated further by developing a thermal model inside the pump to give a more accurate prediction of the oil temperature in these films.

Figure 11 shows the correlation with total efficiency at these four conditions. In general there is good agreement in total efficiency with two exceptions. The first is the overprediction of friction power in the cold temperature with minimum clearances condition causes the model to underpredict the total efficiency. The second is at hot temperature with minimum clearances, the underprediction of flow rate at low speed also leads to an underprediction of total efficiency.

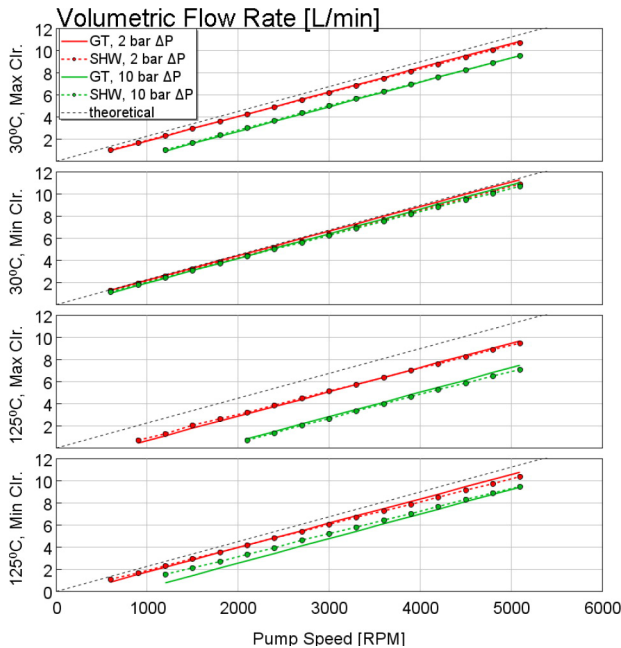


Figure 8. Flow rate comparison between model and experiment. From top to bottom: 30°C - max clearances, 30°C - min clearances, 125°C - max clearances, 125°C - min clearances.

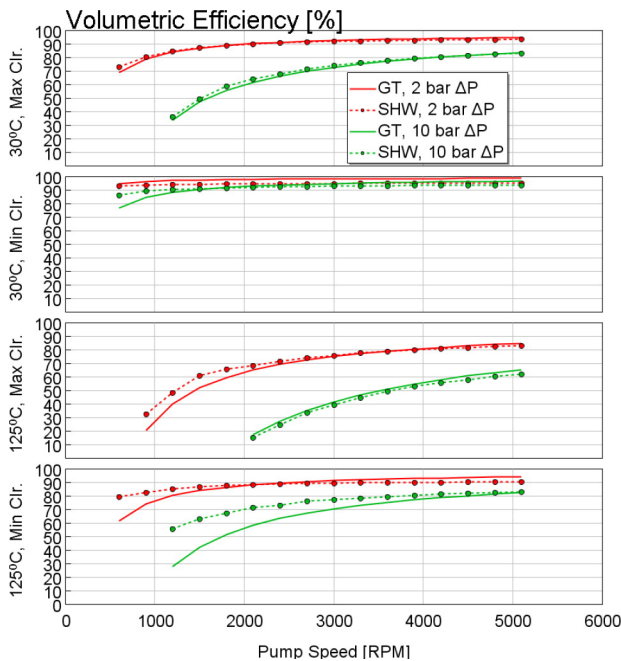


Figure 9. Volumetric efficiency comparison between model and experiment. From top to bottom: 30°C - max clearances, 30°C - min clearances, 125°C - max clearances, 125°C - min clearances.

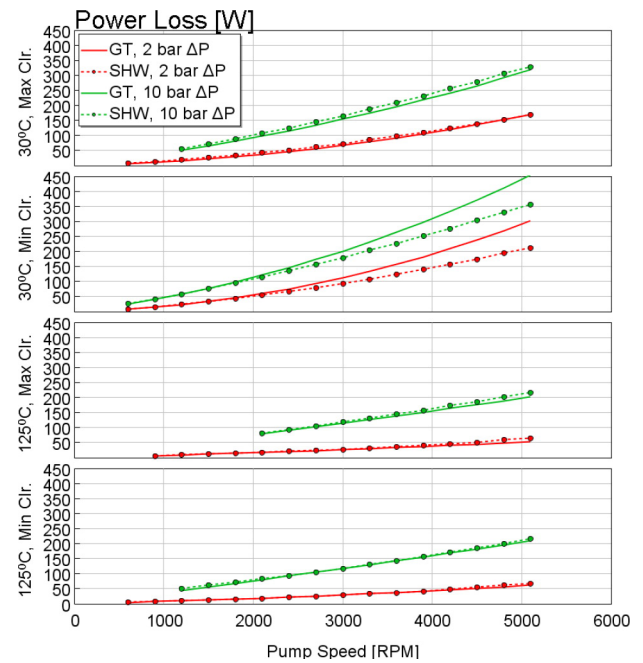


Figure 10. Power loss comparison between model and experiment. From top to bottom: 30°C - max clearances, 30°C - min clearances, 125°C - max clearances, 125°C - min clearances.

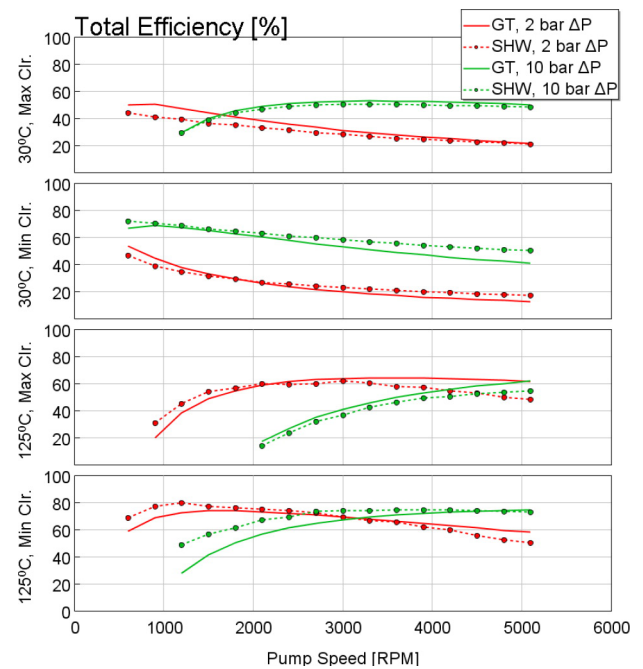


Figure 11. Total efficiency comparison between model and experiment. From top to bottom: 30°C - max clearances, 30°C - min clearances, 125°C - max clearances, 125°C - min clearances.

Figure 12 shows the correlation with mechanical efficiency at these four conditions. The mechanical efficiency is simply the total efficiency divided by the volumetric efficiency.

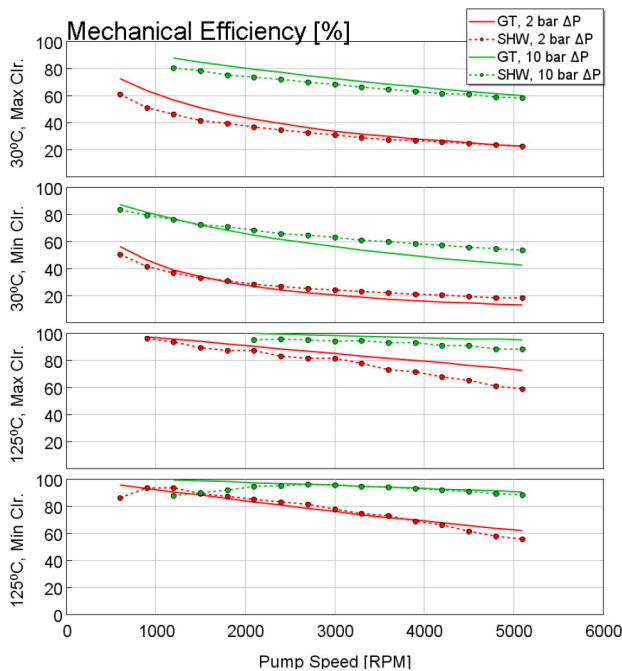


Figure 12. Mechanical efficiency comparison between model and experiment. From top to bottom: 30°C - max clearances, 30°C - min clearances, 125°C - max clearances, 125°C - min clearances.

Power Distribution

The power distribution at 2 bar and 10 bar outlet pressure is broken down in Figure 12 and Figure 13, respectively. At low temperature viscous friction is a significant contributor to the total power requirement of the pump. In fact when clearances are minimum, friction power is more than half the total power loss. Of the four friction components, the outer gear sides appears to be the strongest contributor, followed closely by the outer gear radial gap. At high temperature, viscous friction is only a small contribution to the total power requirement, which is dominated heavily by the fluid power requirement.

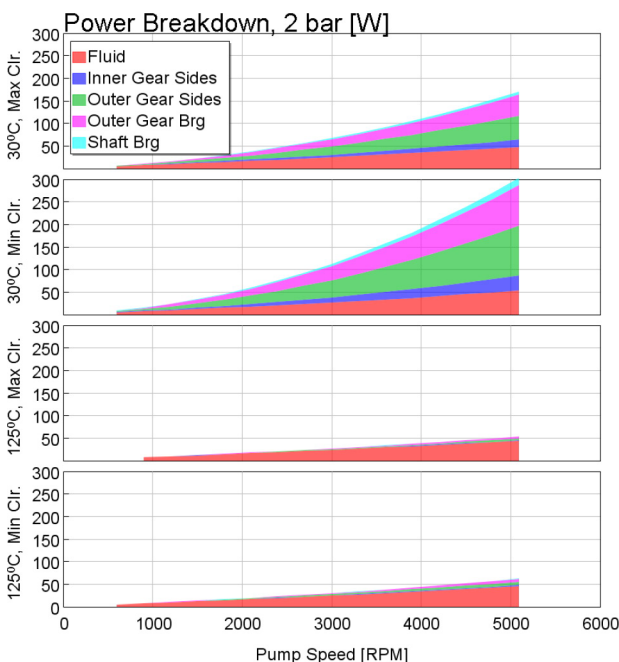


Figure 13. Power distribution at 2 bar. From top to bottom: 30°C - max clearances, 30°C - min clearances, 125°C - max clearances, 125°C - min clearances.

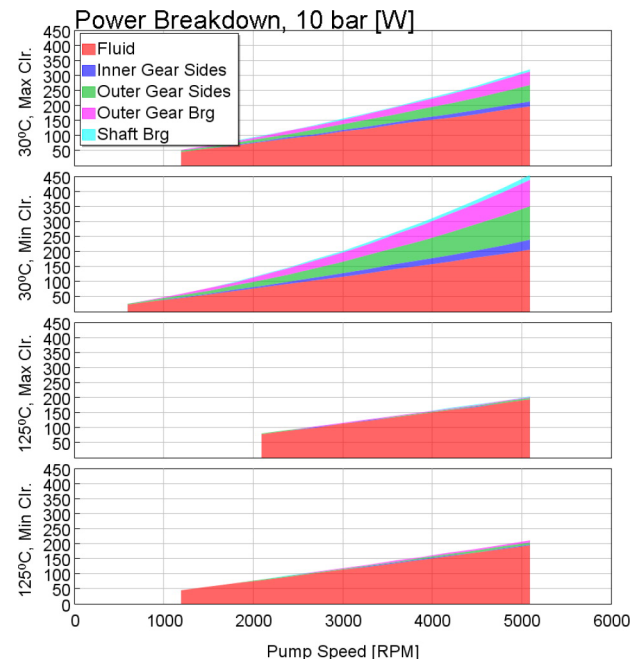


Figure 14. Power distribution at 10 bar. From top to bottom: 30°C - max clearances, 30°C - min clearances, 125°C - max clearances, 125°C - min clearances.

Tradeoff Between Cost and Efficiency

By modeling the pump with a 1D tool, the engineer can easily analyze tradeoffs between cost and efficiency associated with clearance tolerances. This can clearly be seen in Figure 14, for the typical cold engine start operating condition, where the benefits of a higher total efficiency are seen with smaller clearances, but small clearances might mean a prohibitive cost to manufacture. By modeling in 1D, engineers can predict performance trends and make decisions on tolerances early in the design phase that will meet the highest efficiency target while keeping cost to manufacture reasonable.

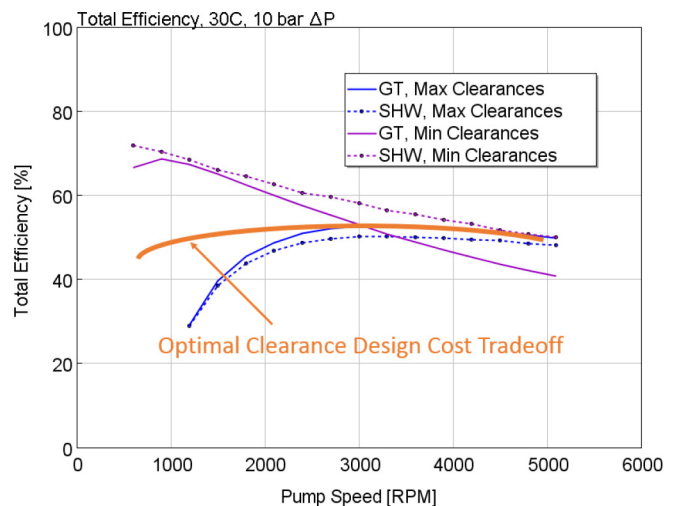


Figure 15. Spread in total efficiency between minimum and maximum clearances.

SUMMARY/CONCLUSIONS

In conclusion, a 1D model has been developed to correlate to experimental gerotor pump performance including flow rate, power loss, volumetric efficiency, and total efficiency. Overall, the model correlates well with experiment, and the following points can be drawn:

1. Gerotor pumps can easily be modeled in 1D with GT-SUITE, through the use of automated model building directly from the CAD model of the pump.
2. An internal leakage model has been developed which takes into account the physical gap between the inner and outer gear, for prediction of the leakage and volumetric efficiency.
3. A basic friction model has been developed to allow the total efficiency to be predicted, with good correlation to experiment.
4. 1D models can be used to quickly give engineers an estimation of the volumetric and total efficiency early in the design stage, and ultimately find the best tradeoff of cost vs. clearance tolerance.

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ACKNOWLEDGMENTS

The authors wish to thank SHW for allowing their pump performance data to be published.

DEFINITIONS/ABBREVIATIONS

$Q_{orifice}$ - Volume flow rate through the orifice
 C_d - Discharge coefficient
 A - Cross-sectional area of leakage
 P_{1r} - Total pressure upstream of the orifice
 P_2 - Static pressure downstream of the orifice
 ρ - Density of the fluid
 $Q_{Poiseuille}$ - Volume flow rate of Poiseuille leakage term
 W - Gear width
 ΔP - Pressure difference across the orifice
 C_{IGOG} - Clearance gap between inner and outer gear
 μ - Dynamic viscosity
 L - Length of leakage path
 Q_{leak} - Volume flow rate of leakage
 C_f - Friction factor
 A_s - Surface area of leakage
 c_p - Fluid specific heat capacity
 A_{IG} - Surface area of the film between the inner gear and pump housing (one side of the inner gear)
 ω_{IG} - Angular velocity of the inner gear
 R_{IG} - Mean radius of friction surface of inner gear with respect to inner gear center
 C_{IG} - Clearance gap between inner gear and pump housing
 A_{OG} - Surface area of the film between the outer gear and pump housing (one side of the outer gear)
 ω_{OG} - Angular velocity of the outer gear
 R_{OG} - Mean radius of friction surface of outer gear with respect to outer gear center
 C_{OG} - Clearance gap between outer gear and pump housing
 R_{OGr} - Radius outer gear with respect to inner gear center
 C_{OGB} - Radial clearance gap between the outer gear and housing
 W_{brg} - Width of bearing support
 R_{brg} - Radius of bearing support
 C_{brg} - Radial clearance of bearing
 P_{total} - Total power of the pump
 $P_{friction}$ - Total friction power
 P_{fluid} - Fluid power of the pump